# Exam advanced logic, 2015, 16th of June

## Model answers

1. Basis step: Take an arbitrary propositional atom $p$.

- Let $v$ be a valuation such that $v(p)=1$. Then $p$ is satisfiable
- Let $v'$ be a valuation such that $v(p)=0$. Then $\neg p$ is satisfiable

Inductive hypothesis (IH): Let $A$ and $B$ be arbitrary formulas of $\mathcal{L}_E$. Suppose that:
Both $A$ and $\neg A$ are satisfiable and
Both $B$ and $\neg B$ are satisfiable.

Inductive step

Case 1: Consider $\neg A$. By the inductive hypothesis, $\neg A$ is satisfiable. Consider $\neg\neg A$. This is logically equivalent to $A$, which is satisfiable by the inductive hypothesis

Case 2: Suppose that $\pi(A) \cap \pi(B) = \emptyset$. Then $A \equiv B$ and $\neg(A \equiv B)$ are formulas of $\mathcal{L}_E$.
Consider $A \equiv B$. Let $v$ be a valuation such that $v(A)=1$ and let $v'$ be a valuation such that $v(B)=1$.
Such valuations exist by the inductive hypothesis.
Now consider the following valuation $v''$:

$$v''(p) = \begin{cases} v(p) & \text{if } p \in \pi(A) \\ v'(p) & \text{otherwise} \end{cases}$$

It is clear that $v''(A)=1$ and $v''(B)=1$ since $\pi(A) \cap \pi(B) = \emptyset$. So, $v''(A \equiv B) = 1$.
Therefore $A \equiv B$ is satisfiable.
Consider $\neg(A \equiv B)$. Let $v$ be such that $v(A)=1$ and let $v'$ be such that $v(\neg B)=1$ (both by IH).
Define $v''$ from $v$ and $v'$ similarly as above.
Then $v''(A \equiv B) = 0$, so $v''(\neg(A \equiv B)) = 1$ and $\neg(A \equiv B)$ is satisfiable.

By induction, for all formulas $A$ of $\mathcal{L}_E$, $A$ and $\neg A$ are both satisfiable

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| $p$ | $q$ | $p \ni q$ | $\neg p$ | $\supset q$ | $q$ |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | i | i | 0 | 1 | i |
| 1 | 0 | 0 | 0 | 1 | 0 |
| i | 1 | 1 | i | 1 | 1 |
| i | i | i | i | i | i |
| i | 0 | i | i | i | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | i | 1 | 1 | i | i |
| 0 | 0 | 1 | 1 | 0 | 0 |

When $v(p)=i$ and $v(q)=0$, both premises have a designated value, but the conclusion does not. Therefore the inference is not valid in LP.

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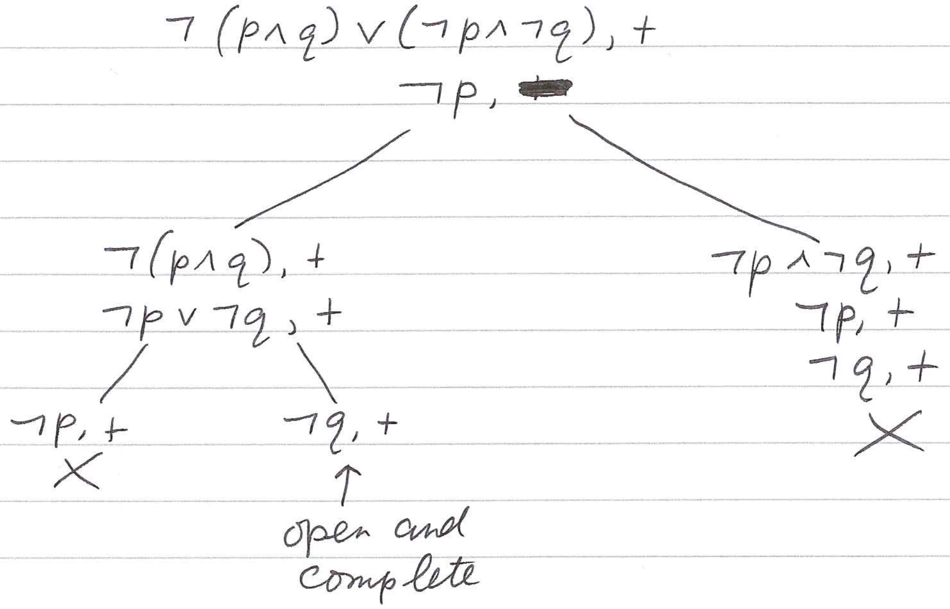
3. Question: $\neg(p\wedge q)\vee(\neg p\wedge\neg q) \not\models_{K_3} \neg p$ ?



The tableau has an open, complete branch, so the inference is not valid.
We can read off a countermodel from the branch: $\begin{cases} q\rho 0 \\ \text{nothing obtains about } p \end{cases}$

(We also count correct: $p\rho 1$ ; but not 'The value of $p$ is arbitrary' because it is explicitly ruled out by "¬p, [redacted]" that $p\rho 0$)

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4. Question: $p \to q,\ q \to r \models_{0.7} p \to r$ ?

Take a valuation $v$ such that:

$v(p) = 1$

$v(q) = 0.7$

$v(r) = 0.4$

Consider $p \to q$. Since $v(p) > v(q)$, $v(p \to q) = 1 - v(p) + v(q) = 0.7$

Consider $q \to r$. Since $v(q) > v(r)$, $v(q \to r) = 1 - v(q) + v(r) = 0.7$

Consider $p \to r$. Since $v(p) > v(r)$, $v(p \to r) = 1 - v(p) + v(r) = 0.4$

So the premises $p \to q$ and $q \to r$ have a designated value, but the conclusion does not.

Hence,

$$p \to q,\ q \to r \not\models_{0.7} p \to r$$

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Question: $\Box p \supset \Box(\Box p \wedge p) \vdash_K \Box p \supset \Box\Box p$ ?

$\Box p \supset \Box(\Box p \wedge p), 0$

$\neg(\Box p \supset \Box\Box p), 0$

$\Box p, 0$

$\neg\Box\Box p, 0$

$\Diamond\neg\Box p, 0$

$0r1$

$\neg\Box p, 1$

$p, 1$

$\neg\Box p, 0$

X

$\Box(\Box p \wedge p), 0$

$\Box p \wedge p, 1$

$\Box p, 1$

$p, 1$

X

The tableau closes, so the inference is valid.

Exam Advanced Logic - Qu: $[F]p \overset{?}{\vdash}_{K^{t}_{\eta\delta}} \langle F\rangle\langle F\rangle p$

6. $[F]p, 0$
$\neg\langle F\rangle\langle F\rangle p, 0$
$[F]\neg\langle F\rangle p, 0$
$0r1$ $\eta$
$p, 1$
$\neg\langle F\rangle p, 1$
$[F]\neg p, 1$
$1r2$ $\eta$
$\neg p, 2$
$0r3$ $\delta$
$3r1$
$p, 3$
$\neg\langle F\rangle p, 3$
$[F]\neg p, 3$
$\neg p, 1$
$\times$

The tableau closes, so the inference is valid.

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7 Take an arbitrary world $w_i \in W$ such that $i$ occurs on $b$. Since $b$ is complete, it must be the case that $iri$ occurs on $b$. Therefore $w_i R w_i$. Therefore $R$ is reflexive.

Take three arbitrary worlds $w_i, w_j, w_k \in W$, such that $i$, $j$ and $k$ occur on $b$. Suppose that $w_i R w_j$ and $w_j R w_k$. Since $I$ is induced by $b$, it must be the case that $irj$ and $jrk$ occur on $b$. Therefore $irk$ occurs on $b$, and so $w_i R w_k$. Therefore $R$ is transitive.

8. $\exists x Px \supset \exists x \Box Px, 0$

$\neg(\exists x Px \supset \Box \exists x Px), 0$

$\exists x Px, 0$

$\neg \Box \exists x Px, 0$

$\Diamond \neg \exists x Px, 0$

$0r1$

$\neg \exists x Px, 1$

$\forall x \neg Px, 1$

$\neg \exists x Px, 0$ $\qquad$ $\exists x \Box Px, 0$

$\times$ $\qquad$ $\mathcal{E}c, 0$

$Pc, 0$

$\mathcal{E}d, 0$

$\Box Pd, 0$

$Pd, 1$

$\neg \mathcal{E}d, 1$ $\qquad$ $\neg Pd, 1$

$\neg \mathcal{E}c, 1$ $\qquad$ $\neg Pc, 1$ $\qquad$ $\times$

There is an open, complete branch. Therefore, the inference is invalid.

$W = \{w_0, w_1\}$

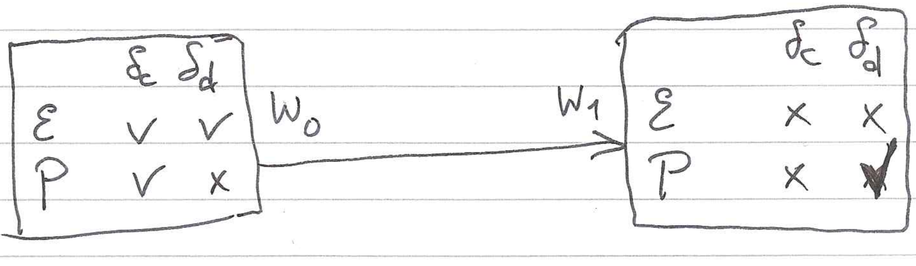
$R = \{\langle w_0, w_1 \rangle\}$

$D_{w_0} = \{\delta_c, \delta_d\}$

$D_{w_1} = \emptyset$

$v_{w_0}(P) = \{\delta_c\}$

$v_{w_1}(P) = \{\delta_1\}$



|  | $\delta_c$ | $\delta_d$ |
|---|---|---|
| $\mathcal{E}$ | ✓ | ✓ |
| P | ✓ | × |

|  | $\delta_c$ | $\delta_d$ |
|---|---|---|
| $\mathcal{E}$ | × | × |
| P | × | ✓ |

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9. a) (i). • $\emptyset$ is a process because "for all defaults in $\emptyset$" holds in an empty way
  - $\emptyset$ is not closed because there are still defaults applicable to it (e.g. $\delta_1$ and $\delta_3$)
  - $Out(\emptyset) = \emptyset$ so certainly $In(\emptyset) \cap Out(\emptyset) = \emptyset$, so $\emptyset$ is successful

(ii). • $(\delta_1)$ is a process because $\delta_1$ is applicable to $Th(W)$: $B(d) \in Th(W)$ and $\neg\neg A(d) \notin Th(W)$
  - $(\delta_1)$ is closed because $\delta_2$ is not applicable ($A(d) \notin Th\{B(d), G(d), \neg A(d)\}$); and $\delta_3$ is not applicable because $\neg A(d) \in In(\delta_1)$.
  - $(\delta_1)$ is successful because $In(\delta_1) \cap Out(\delta_1) = Th(\{B(d), G(d), \neg A(d)\}) \cap \{A(d)\} = \emptyset$

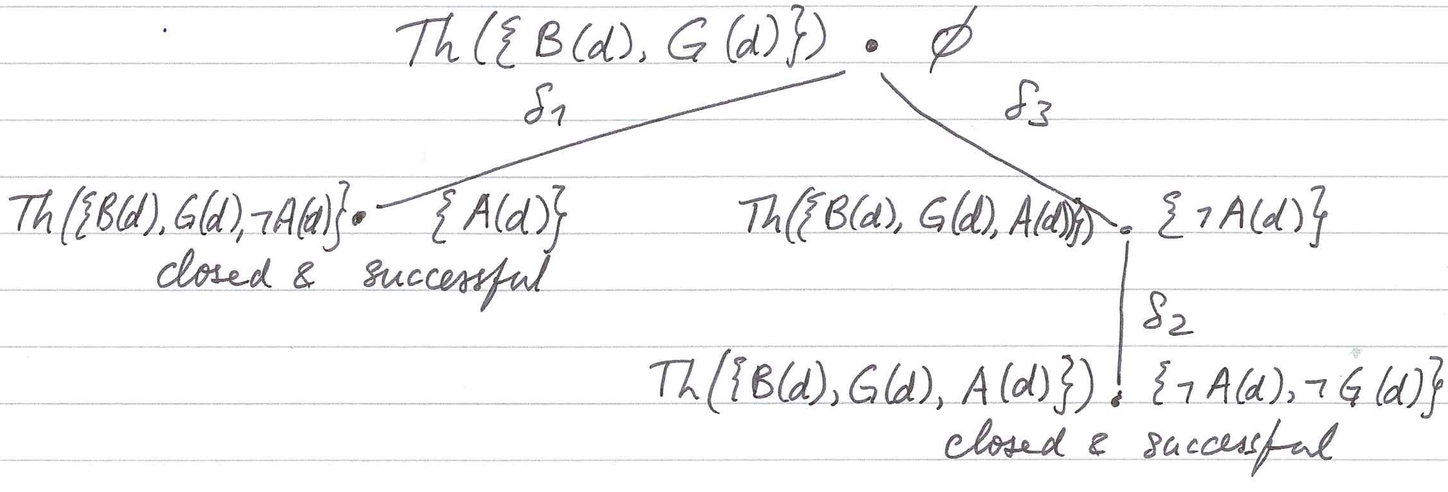
(iii). • $(\delta_2)$ is not a process, because $A(d) \notin Th(W)$, so $\delta_2$ is not applicable to $In(\emptyset)$.

(iv). • $(\delta_3)$ is a process: $G(d) \in Th(W)$ and $\neg(A(d) \notin Th(W)$, so $\delta_3$ is applicable to $In(\emptyset)$.
  - $(\delta_3)$ is not closed, because $\delta_2$ is applicable to it: $A(d) \in Th(\{B(d), G(d), A(d)\}) = In(\delta_3)$, and $\neg G(d) \notin Th(\{B(d), G(d), A(d)\}) = In(\delta_3)$.
  - $(\delta_3)$ is successful because $In(\delta_3) \cap Out(\delta_3) = Th(\{B(d), G(d), A(d)\}) \cap \{\neg A(d)\} = \emptyset$

(v) • $(\delta_3, \delta_2)$ is a process. See (IV), + $\delta_2$ is applicable to $In(\delta_3)$ (see IV)
  - $(\delta_3, \delta_2)$ is closed because $\delta_1$ is not applicable to $In(\delta_3, \delta_2) = Th(\{B(d), G(d), A(d)\})$, which contains $\neg\neg A(d)$
  - $(\delta_3, \delta_2)$ is successful because $In(\delta_3, \delta_2) \cap Out(\delta_3, \delta_2) = Th(\{B(d), G(d), A(d)\}) \cap \{\neg A(d), \neg G(d)\} = \emptyset$

(VI). • $(\delta_3, \delta_2, \delta_1)$ is not a process, because $\delta_1$ is not applicable to $In(\delta_3, \delta_2)$ (see V)

9 (b) Process Tree :



(c) The extensions are :

$Th(\{B(d), G(d), \neg A(d)\})$

$Th(\{B(d), G(d), A(d)\})$